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ON DISCRETE INTERACTION OF A PLATE AND A DAMAGED STRINGER*

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There is investigated the plane contact problem for an elastic system consisting of an unbounded plate reinforced by an infinite homogeneous stringer in terms of periodically located hard circular inclusions (rivets). There are considered to modifications of a single damage of the stringer, that is modeled by a compressively strained rod: on a section between rivets, and on a section passing through the center of the rivets. In both cases the problem reduces to an infinite system of linear algebraic equations in the axial components (projections on the stringer axis) of the reactive forces transmitted by the rivets. The exact solution of such systems is constructed in quadratures by using the Laurent transformation and the apparatus of the Riemann—Hilbert boundary value problem. The results obtained can be used in estimating the residual strength of damaged riveted panels. Some problems concerning the discrete interaction of a plate and an undamaged bar are studied in /1—5/.

1. Formulation of the problem. Let us consider a plane elastic system formed by an unbounded thin plate and an infinite homogeneous stringer fastened to the plate by periodically arranged rivets. It is assumed that the stringer is fractured once in some section and the external loads acting on the system are represented by constant forces at infinity in the plate (homogeneous external field of plate loads), and by concentrated forces applied to the centers of the rivets in the stringer (Fig.1).

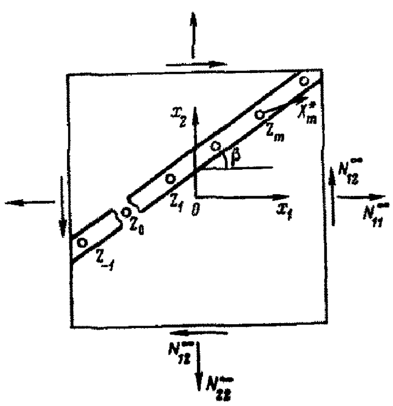

Fig.1

Let us make the assumptions. 1°. Plane state of stress conditions are realized in the plate. 2°. The stringer is modeled by a bar operating only under tension-compression; its weakening due to riveting is not taken into account. 3°. The rivets in the plate are simulated by stiff circular inclusions fastened to the plate along their contact surface; the radius of the inclusions is small compared to their spacing, and the centers of the inclusions are on the stringer axis. 4°. The stringer interacts with the plate at its middle surface by means of the inclusions. This latter assumption excludes from consideration the eccentricity of stringer attachment to the plate relative to its middle surface, and the friction force between the plate and the stringer.

Let us introduce the notation: ν, E are, respectively, the Poisson's ratio and Young's modulus of the plate material, A is the stringer stiffness under tension—compression, h is the plate thickness, R is the spacing between the centers of adjacent inclusions (the rivet spacing), and r is the radius of the inclusions. The remaining symbols introduced below are considered dimensionless: quantities with a linear dimensionality, linear forces in the plate, forces in the stringer and the concentrated forces applied to the centers of the inclusions are referred, respectively, to R, $Eh/(1+\nu)$, and A and $8\pi EhR/(1+\nu)^2$.

Let an elastic system be referred to a rectangular Cartesian x_1, x_2 coordinate system located in the plate middle surface, and let $z = x_1 + ix_2$ be a complex variable $(i=\sqrt{-1})$. Simultaneously assuming the stringer not ruptured, we imagine it separated from the plate (evidently cutting the inclusion out), and we apply the unknown interaction forces $X_m = X_{1m} + iX_{2m}$ and $-X_m$, respectively, to the centers $z_m = z_0 + me^{i\beta}$ $(m = 0, \pm 1, \pm 2, \ldots)$ of the inclusions in the plate and stringer. Here β is the stringer slope to the x_1 axis. The field of elastic displacements and linear forces in the isolated plate is determined by the Kolosov—Muskhelishvili formulas /6/:

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$$w(z) = u_1 + iu_2 = \varkappa\varphi(z) - \overline{z\varphi'(z)} - \overline{\psi(z)} \tag{1.1}$$
$$N_{11} + N_{22} = 2\left[\varphi'(z) + \overline{\varphi'(z)}\right]$$
$$N_{22} - N_{11} + 2iN_{12} = 2\left[\bar{z}\varphi''(z) + \psi'(z)\right], \quad \varkappa = (3 - \nu)/(1 + \nu)$$

Here u_α is the displacement along the x_α axis $(\alpha = 1, 2)$, $N_{\alpha\beta}$ $(\alpha, \beta = 1, 2)$ are the Cartesian components of the linear force tensor, $\varphi(z)$ and $\psi(z)$ are complex potentials, generally determined from the solution of the plane problem of elasticity theory for a plane with a periodic system of stiff circular inclusions. The assumption about the smallness of the rivet radius compared to their spacing (in practice $\varepsilon = r/R = 0.02 - 0.1$) permits us to limit ourselves to taking account of the mutual influence of the inclusions asymptotically by using the superposition principle in the form /6/:

$$\varphi(z) = \varphi_*(z) + \sum_{m=-\infty}^{\infty} \varphi_m(z), \quad \psi(z) = \psi_*(z) + \sum_{m=-\infty}^{\infty} \psi_m(z) \tag{1.2}$$

$$\varphi_*(z) = \Gamma z, \quad \psi_*(z) = \Gamma' z \tag{1.3}$$
$$\varphi_m(z) = -X_m \ln(z - z_m)$$
$$\psi_m(z) = \varkappa\bar{X}_m \ln(z - z_m) + X_m\left[\frac{\bar{z}_m}{z - z_m} + \frac{\varepsilon^2}{(z - z_m)^2}\right]$$
$$(m = 0, \pm 1, \pm 2, \ldots; \quad 4\Gamma = N_{11}^\infty + N_{22}^\infty;$$
$$2\Gamma' = N_{22}^\infty - N_{11}^\infty + 2iN_{12}^\infty)$$

Here $\varphi_*(z)$, $\psi_*(z)$ are the potentials for a plane without inclusions due to a homogeneous external field of loads given at infinity by tensor component of the constant forces $N_{\alpha\beta}^\infty$ $(\alpha, \beta = 1, 2)$, and $\varphi_m(z)$, $\psi_m(z)$ are potentials for a plane with just an m-th circular inclusion to whose center the force X_m is applied.

The static conjugate conditions of adjacent sections of an isolated stringer at the points z_m $(m = 0, \pm 1, \pm 2, \ldots)$ have the form

$$X_m = X_m^* + \omega(N_m - N_{m-1})e^{i\beta}, \quad \omega = \frac{(1+\nu)^2 A}{8\pi EhR} \tag{1.4}$$

where X_m^* is the complex vector of the external force acting on the stringer at the center of the rivet m, N_m is the force in the stringer at the section m (between the rivets m and $m+1$), and ω is the stiffness parameter of the elastic system.

By using the relationships

$$X_m e^{-i\beta} = P_m + iQ_m, \quad X_m^* e^{-i\beta} = P_m^* + iQ_m^*$$

we, respectively, introduce the axial (P_m, P_m^*) and transverse (Q_m, Q_m^*) components of the forces X_m and X_m^*. According to (1.4), $Q_m = Q_m^*$, and

$$P_m = P_m^* + \omega(N_m - N_{m-1}) \tag{1.5}$$

In the undamaged system, the conditions for combined strain of the plate and stringer will be satisfied if it is required that the equality

$$\operatorname{Re}\{[w(z_{m+1}) - w(z_m)]e^{-i\beta}\} = N_m$$

would be valid for all integer m.

Transforming its left side by using the formulas (1.1)—(1.5), and taking account of the smallness of ε we find

$$\gamma - \sum_{n=-\infty}^{\infty} \Gamma_{m-n} P_n = N_m \tag{1.6}$$

$$\Gamma_0 = -\Gamma_{-1} = -1 + \varepsilon^2 - 2\varkappa \ln \varepsilon \tag{1.7}$$
$$\Gamma_n = -\Gamma_{-n-1} = 2\varkappa \ln(1 + n^{-1}) - \varepsilon^2 \frac{2n+1}{n^2(n+1)^2} \quad (n \neq 0, -1)$$
$$2\gamma = \frac{1-\nu}{1+\nu}(N_{11}^\infty + N_{22}^\infty) + (N_{11}^\infty - N_{22}^\infty)\cos 2\beta + 2N_{12}^\infty \sin 2\beta$$

For all integer m the set of relationships (1.5) and (1.6) form an infinite system of linear algebraic equations in the required quantities N_m and P_m. It is later convenient to take the reactive forces P_m as the fundamental unknowns. Eliminating N_m from the relationships mentioned, we see that the system of equations

$$P_m + \omega \sum_{n=-\infty}^{\infty} B_{m-n}P_n = P_m^* \quad (m = 0, \pm 1, \pm 2, \ldots) \tag{1.8}$$

$$B_0 = 2\Gamma_0 = -2(1 - \varepsilon^2 + 2\varkappa \ln \varepsilon), \qquad B_1 = \Gamma_1 + \Gamma_{-1} = 1 - 1.75\varepsilon^2 + 2\varkappa \ln 2\varepsilon \tag{1.9}$$

$$B_n = \Gamma_n + \Gamma_{-n} = 2\varkappa \ln(1 - n^{-2}) + 2\varepsilon^2 \frac{3n^2 - 1}{n^2(n^2 - 1)^2} \quad (|n| \neq 0, 1)$$

determines P_m.

Formulation of the problem of discrete interaction of the plate and an undamaged stringer is thus completed.

Turning to the formulation of an analogous problem for a single damaged stringer, we extract two, in principle, possible versions in its damage. In the first version, we assume the stringer to be ruptured at a section passing through the center of some rivet, whereupon the coupling of the stringer to this rivet is lost (Fig.1). In the second version the ruptured section of the stringer is between rivets and does not affect coupling of the stringer to the rivets.

Let us first examine the first version of stringer damage. For definiteness, we consider the ruptures section to pass through the center of the zero rivet. In this case (1.6) are valid for all integer $m \neq 0, -1$. As regards the conjugate conditions (1.5), then we should set $N_{-1} = N_0 = 0$ therein. Eliminating the forces N_m which are different from zero from the system (1.5), (1.6) (taking the stipulations made into account), we find

$$P_m + \omega \sum_{n=-\infty}^{\infty} B_{m-n}P_n = P_m^* \quad (m = \pm 2, \pm 3, \ldots) \tag{1.10}$$

$$P_1 + \omega \sum_{n=-\infty}^{\infty} \Gamma_{1-n}P_n = P_1^* + \omega\gamma \tag{1.11}$$

$$P_{-1} - \omega \sum_{n=-\infty}^{\infty} \Gamma_{-2-n}P_n = P_{-1}^* - \omega\gamma, \qquad P_0 = P_0^*$$

In studying the second damage version, we assume the stringer to be ruptured over some section of the part -1. The compatibility conditions (1.6) remain valid for all integer $m \neq -1$ for such a realization of the damage, and the dependences (1.5) are supplemented by the evident equality $N_{-1} = 0$. Eliminating the forces $N_m\ (m \neq -1)$ from these equations, we obtain

$$P_m + \omega \sum_{n=-\infty}^{\infty} B_{m-n}P_n = P_m^* \quad (m = 1, \pm 2, \pm 3, \ldots) \tag{1.12}$$

$$P_0 + \omega \sum_{n=-\infty}^{\infty} \Gamma_{-n}P_n = P_0^* + \omega\gamma \tag{1.13}$$

$$P_{-1} - \omega \sum_{n=-\infty}^{\infty} \Gamma_{-2-n}P_n = P_{-1}^* - \omega\gamma$$

Just as the system (1.8) for the undamaged stringer, the systems (1.10), (1.11) and (1.12), (1.13) permit finding the reactive forces P_m for the appropriate versions of stringer damage. In all cases the elastic field of forces and displacements in the plate are determined from them in conformity with (1.1)—(1.3) while the non-zero forces in the stringer are determined from (1.5).

2. Solution of infinite algebraic systems. Let us examine the formal expansions

$$B(z) = \sum_{m=-\infty}^{\infty} B_m z^m, \quad \Gamma(z) = \sum_{m=-\infty}^{\infty} \Gamma_m z^m$$

that are Laurent transformations of the infinite sequences $\{B_m\}$ and $\{\Gamma_m\}$, respectively. It can be shown that these functions are regular only on the unit circle C and, in conformity with (1.7), (1.9), a dependence

$$B(\zeta) = (1 - \zeta)\Gamma(\zeta), \quad (\zeta \in C)$$

exist between them.

Let us assume that the functions

$$P(z) = \sum_{m=-\infty}^{\infty} P_m z^m, \quad P^*(z) = \sum_{m=-\infty}^{\infty} P_m^* z^m$$

are regular in, at least, the unit circle C. For $P(z)$ this assumption is justified by the solution of the problem. As regards $P^*(z)$, then the customary regularity condition is not restrictive since the regularity of $P^*(z)$ in a ring containing C follows practically always from the given distribution of forces P_m^*. By the definition of the inverse Laurent transform

$$P_m = \frac{1}{2\pi i} \int_C P(\zeta) \zeta^{-m-1} d\zeta \quad (m = 0, \pm 1, \pm 2, \ldots) \tag{2.1}$$

Analogous relationships hold for P_m^* and $P^*(z)$.

Let us note that integration over the contour C is performed counter-clockwise in (2.1) and below.

In the case of an undamged stringer, by using relationships of the type (2.1), the system (2.1) is reduced to the form

$$\frac{1}{2\pi i} \int_C [G(\zeta) P(\zeta) - P^*(\zeta)] \zeta^{-m-1} d\zeta = 0 \tag{2.2}$$

$$G(\zeta) = 1 + \omega B(\zeta) = 1 + \omega(1 - \zeta)\Gamma(\zeta) \quad (\zeta \in C) \tag{2.3}$$

where the exponent m runs through a whole set of integers. Hence, it can be written

$$P(\zeta) = G^{-1}(\zeta) P^*(\zeta) \,.\, (\zeta \in C)$$

Substituting this expression into (2.1), we find that the solution of the system (1.8) has the form

$$P_m = \sum_{n=-\infty}^{\infty} g_{m-n} P_n^* \quad (m = 0, \pm 1, \pm 2, \ldots) \tag{2.4}$$

The quantities g_n here are evaluated from the formulas

$$g_n = \frac{1}{2\pi i} \int_C \frac{\zeta^{-n-1} d\zeta}{G(\zeta)} = \frac{1}{\pi} \int_0^{\pi} \frac{\cos n\sigma}{g(\sigma)} d\sigma \quad (n = 0, \pm 1, \pm 2, \ldots) \tag{2.5}$$

$$g(\sigma) = G(e^{i\sigma}) = 1 + 2\omega \left\{ (\cos\sigma - 1) \left[1 + 2\varkappa \ln 2\varepsilon + \varepsilon^2 \left(\frac{\sigma^2}{2} - \pi\sigma + \frac{\pi^2}{3} \right) \right] + 2\varkappa \sum_{m=2}^{\infty} \ln(1 - m^{-2})(\cos m\sigma - 1) \right\}, \quad (\sigma \in [0, \pi])$$

We turn to the construction of the solution of the system (1.1), (1.11). Just as the system (1.8), by using relationships of the type (2.1), the infinite subsystem (1.10) is reduced to (2.2), which is valid for any integers $m \neq 0, \pm 1$. All these equations will be satisfied if

$$P(\zeta) = P^+(\zeta) + P^-(\zeta) = G^{-1}(\zeta) [P^*(\zeta) + a_1\zeta + a_0 + a_{-1}\zeta^{-1}] \quad (\zeta \in C) \tag{2.6}$$

Here $P^+(z)$ and $P^-(z)$ are, respectively, the regular and principal parts of the function $P(z)$, and a_1, a_0, a_{-1} are still unknown constants which will be determined by the still unused equation (1.11).

By using relationships of the type (2.1), the first two can be represented in the form

$$\frac{1}{2\pi i} \int_C [1 + \omega\Gamma(\zeta)] P(\zeta) \zeta^{-2} d\zeta = P_1^* + \omega\gamma, \quad \frac{1}{2\pi i} \int_C [1 - \omega\zeta\Gamma(\zeta)] P(\zeta) d\zeta = P_{-1}^* - \omega\gamma$$

Substituting (2.6) here with the dependence (2.3) taken into account, we find

$$P^+(1) - P^-(1) = P^{*+}(1) - P^{*-}(1) + 2\omega\gamma - a_1 - a_0 - a_{-1}$$
$$P^+(1) - P^-(1) = P^{*+}(1) - P^{*-}(1) + 2\omega\gamma + a_1 + a_0 + a_{-1}$$

Here $P^{*+}(1)$ and $P^{*-}(1)$ are the limit values, respectively, of the regular $P^{*+}(z)$ and the principal $P^{*-}(z)$ parts of the function $P^*(z)$ at the point $z = 1$.

These latter equations are equivalent to the identities

$$a_1 + a_0 + a_{-1} = 0 \tag{2.7}$$

$$P^+(1) - P^-(1) = P^{*+}(1) - P^{*-}(1) + 2\omega\gamma \tag{2.8}$$

In order to convert (2.8) into an explicit dependence between the required constants, the functions $P^+(z)$ and $P^-(z)$ must be found. In this connection, we note that the expression

$$P(\zeta) = P^+(\zeta) + P^-(\zeta) = G^{-1}(\zeta)\,[P^*(\zeta) + a_0(1 - \zeta^{-1}) + a_1(\zeta - \zeta^{-1})] \quad (\zeta \in C) \tag{2.9}$$

resulting from (2.6) and (2.7) can be treated as the simplest Riemann–Hilbert problem. Its solution that satisfies the evident condition $P^-(\infty) = 0$ has the form /7/

$$P^\pm(z) = \pm\,[p^\pm(z) + a_0\lambda_0^\pm(z) + a_1\lambda_1^\pm(z)] \quad (z \in D_\pm) \tag{2.10}$$

We have here introduced the piecewise-holomorphic functions

$$p^\pm(z) = \frac{1}{2\pi i}\int_C \frac{P^*(\zeta)\,d\zeta}{G(\zeta)(\zeta - z)}, \quad \lambda_0^\pm(z) = \frac{1}{2\pi i}\int_C \frac{(1-\zeta^{-1})\,d\zeta}{G(\zeta)(\zeta - z)} \tag{2.11}$$

$$\lambda_1^\pm(z) = \frac{1}{2\pi i}\int_C \frac{(\zeta - \zeta^{-1})\,d\zeta}{G(\zeta)(\zeta - z)} \quad (z \in D_\pm)$$

and the symbols D_+ and D_- are for domains, respectively, inside and outside the unit circle C.

The Sokhotskii formulas for the functions (2.10) and (2.11) permit reduction of (2.8) to the form

$$a_0 g_0 + a_1(g_0 + g_1) = \frac{1}{2}\,[P^{*+}(1) - P^{*-}(1)] + \omega\gamma - p(1) \tag{2.12}$$

$$2p(1) = p^+(1) + p^-(1) = \frac{1}{\pi i}\int_C \frac{P^*(\zeta)\,d\zeta}{G(\zeta)(\zeta - 1)} = \sum_{m=-\infty}^{\infty} P_m^* \sum_{n=-m}^{m} g_n \tag{2.13}$$

where it is taken into account that (see (2.11) and (2.5))

$$\lambda_0(1) = \frac{\lambda_0^+(1) + \lambda_0^-(1)}{2} = \frac{1}{2\pi i}\int_C \frac{d\zeta}{\zeta G(\zeta)} = g_0, \quad \lambda_1(1) = \frac{\lambda_1^+(1) + \lambda_1^-(1)}{2} = \frac{1}{2\pi i}\int_C \frac{(1+\zeta)\,d\zeta}{\zeta G(\zeta)} = g_0 + g_1$$

We note that according to (2.1) and (2.9)

$$P_m = \sum_{n=-\infty}^{\infty} g_{m-n} P_n^* + a_0(g_m - g_{m+1}) + a_1(g_{m-1} - g_{m+1}), \quad (m = 0, \pm 1, \pm 2, \ldots) \tag{2.14}$$

Hence, taking account of the last equation in (1.11), we obtain

$$a_0 = \frac{1}{g_0 - g_1}\left(P_0^* - \sum_{n=-\infty}^{\infty} g_n P_n^*\right)$$

The constant a_1 is now easily determined from (2.12). Replacing a_0 and a_1 in (2.14) by their obtained expressions, we find that the solution of the system (1.10), (1.11) has the form

$$P_m = \sum_{n=-\infty}^{\infty} g_{m-n} P_n^* + \left(P_0^* - \sum_{n=-\infty}^{\infty} g_n P_n^*\right) \frac{g_0(g_m - g_{m-1}) + g_1(g_m - g_{m+1})}{g_0^2 - g_1^2} + \tag{2.15}$$

$$\left[\frac{P^{*+}(1) - P^{*-}(1)}{2} + \omega\gamma - p(1)\right] \frac{g_{m-1} - g_{m+1}}{g_0 + g_1}, \quad (m = 0, \pm 1, \pm 2, \ldots)$$

The solution of the system (1.12), (1.13) can also be constructed analogously. However, there is no need for this since it can be derived from the preceding reasoning if $a_1 = 0$ is taken therein. The desired solution is a corollary of (2.12) and (2.14) for $a_1 = 0$ and has the form

$$P_m = \sum_{n=-\infty}^{\infty} g_{m-n}P_n^* + \left[\frac{P^{*+}(1) - P^{*-}(1)}{2} + \omega\gamma - p(1)\right]\frac{g_m - g_{m+1}}{g_0} \quad (m = 0, \pm 1, \pm 2, \ldots) \qquad (2.16)$$

By using relationships of the type (2.1) the infinite subsystem (1.12) is indeed again reduced to (2.2) which is valid now for all integers $m \neq 0, -1$. The function (2.6) will evidently satisfy these equations under the condition $a_1 = 0$. Furthermore, it can be seen that by using this function the equations (1.13) are converted, just as the first and second equations of (1.11) into the identities (2.7) (for $a_1 = 0$) and (2.8). Tracing the further behavior of the construction of the solution of the system (1.10), (1.11), we arrive at (2.16).

The solutions of (2.15) and (2.16) show that the calculation of the reactive forces P_m in a system with a damaged stringer would reduce, as in an undamaged system, to the quadratures (2.5). Comparing these solutions with (2.4), we see that the first terms therein correspond to the system with undamaged stringer, while the influence of the damage is described by the subsequent terms.

It should be emphasized that all the preceding reasoning, including the final results (2.15), (2.16), is valid for the kinds of stringer damage stupulated above whose point of location relative to the rivet numbering taken was strictly fixed. This latter does not constrain the generality of the results obtained, which can be extended, as is easily seen, to the case of an arbitrary location of the stringer damage relative to the rivet numbering by a shift in the index. Thus, if the stringer is ruptured at the rivet s, then the solution is obtained from (2.15) after replacing m in its right side by $m - s$. Analogously, in order to find the solution of the problem for a stringer damaged between rivets within a space s, it is sufficient to replace m in the right side of (2.16) by $m - s - 1$.

In conclusion, let us note that the approach elucidated permits the solution of even more complex problems about the discrete interaction of an unlimited plate and a multiply damaged infinite stringer.

3. Some numerical results.

As an illustration, let us consider the case when only the external field of plate loads acts on an elastic system. Then $P_m^* = 0$ $(m = 0, \pm 1, \pm 2, \ldots)$, $p(1) = 0$ (see (2.13)) and according to (2.15), (2.16)

$$\frac{P_m}{\omega\gamma} = \frac{g_{m-1} - g_{m+1}}{g_0 + g_1} \quad (m = 0, \pm 1, \pm 2, \ldots) \qquad (3.1)$$

if the stringer is damaged at the zero rivet and

$$\frac{P_m}{\omega\gamma} = \frac{g_m - g_{m+1}}{g_0} \quad (m = 0, \pm 1, \pm 2, \ldots) \qquad (3.2)$$

if the stringer is ruptured between rivets in the section — 1.

Some results of calculating the reactive forces in the rivets loaded most are represented in Figs.2 for $\nu = 1/3$ and $\varepsilon = 0.1$; the solid lines correspond to (3.1) and the dashes to (3.2). Let us note that under the same loading of an elastic system with an undamaged stringer, the reactive forces at all the rivets are zero (see (2.4)).

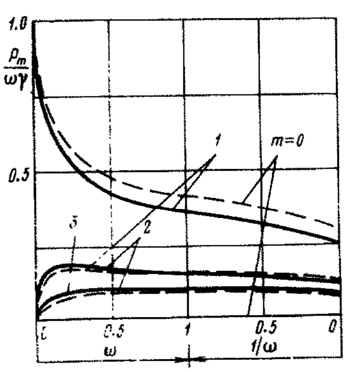

Fig.2

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